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Time Evolution of Density Operator for Field Damping in Squeezed Bath Calculated by Squeezing Transformation and Entangled State Representation

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Abstract We find that a squeezing transformation can efficiently simplify the density operator equation of field damping in a squeezed bath. Then the entangled state representation is introduced to solve the simplified equation and the time evolution of density operator, which turns out to be a mixed coherent squeezed state.

Keywords Entangled state representation · Squeezing transformation · Master equation · Thermo field dynamics

1 Introduction

It is well-known that the standard theory of density operator equation (master equation) for a damped harmonic oscillator with squeezed bath is [1–3]

$$\begin{aligned}\frac{d}{dt}\rho = &-\frac{\lambda}{2}(N+1)[a^{\dagger}a\rho - 2a\rho a^{\dagger} + \rho a^{\dagger}a] \\ &-\frac{\lambda}{2}N[aa^{\dagger}\rho - 2a^{\dagger}\rho a + \rho aa^{\dagger}] + \frac{\lambda}{2}M[aa\rho - 2a\rho a + \rho aa] \\ &+\frac{\lambda}{2}M^{*}[a^{\dagger}a^{\dagger}\rho - 2a^{\dagger}\rho a^{\dagger} + \rho a^{\dagger}a^{\dagger}], \end{aligned} \tag{1}$$

where λ is the damping constant, N and M are the mean number of quanta in thermal and squeezed baths, respectively, satisfying $|M|^2 = N(N+1)$ for a squeezed vacuum reservoir.

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Usually, this operator master equation is converted into an equivalent c-number equation by virtue of P-representation in the coherent state basis [4], which takes a considerable amount of work (see e.g., Scully and Zubairy, Quantum Optics [1]). In this paper we shall reveal that there exists a unitary squeezing transform which can significantly simplify (1) and its corresponding c-number equation. Then the entangled state representation is introduced to solve the simplified equation and the time evolution of density operator, which turns out to be a mixed coherent squeezed state.

2 The Squeezing Transformation and the Reduced Master Equation

By introducing two complex parameters $f = |f|e^{-i\varphi}$ and $g = |g|e^{-i\phi}$, and $|f|^2 + |g|^2 = 1$, and noting $|M|^2 = N(N+1)$, we can identify

$$N \equiv \sinh^2\psi = \frac{4|f|^2|g|^2}{(|f|^2 - |g|^2)^2}, \qquad N + 1 = \cosh^2\psi = \frac{1}{(|f|^2 - |g|^2)^2}, \tag{2}$$

$$M \equiv \sinh\psi\cosh\psi e^{-i\theta} = \frac{2|f||g|}{(|f|^2 - |g|^2)^2} e^{-i(\varphi+\phi)}, \tag{3}$$

here $\theta = \varphi + \phi$. Substituting (2) and (3) into (1) we see that the latter can be re-combined according to the sequence of ρ in each term, i.e.,

$$\begin{aligned}
\frac{d}{dt}\rho &= \frac{\lambda}{2(|f|^2 - |g|^2)^2}\{-[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a] - 4|f|^2|g|^2[aa^\dagger\rho - 2a^\dagger\rho a + \rho aa^\dagger] \\
&\quad + 2|f||g|e^{-i\theta}[aa\rho - 2a\rho a + \rho aa] + 2|f||g|e^{i\theta}[a^\dagger a^\dagger\rho - 2a^\dagger\rho a^\dagger + \rho a^\dagger a^\dagger]\} \\
&= \frac{\lambda}{2(|f|^2 - |g|^2)^2}[2(a - 2|f||g|e^{-i\theta}a^\dagger)\rho(a^\dagger - 2|f||g|e^{i\theta}a) \\
&\quad - (a^\dagger - 2|f||g|e^{i\theta}a)(a - 2|f||g|e^{-i\theta}a^\dagger)\rho \\
&\quad - \rho(a^\dagger - 2|f||g|e^{i\theta}a)(a - 2|f||g|e^{-i\theta}a^\dagger)].
\end{aligned} \tag{4}$$

Further, by defining

$$\begin{aligned}
A &= \frac{1}{|f|^2 - |g|^2}(a - 2|f||g|e^{-i\theta}a^\dagger) = a\cosh\psi - a^\dagger\sinh\psi e^{-i\theta}, \\
A^\dagger &= \frac{1}{|f|^2 - |g|^2}(a^\dagger - 2|f||g|e^{i\theta}a) = a^\dagger\cosh\psi - a\sinh\psi e^{i\theta},
\end{aligned} \tag{5}$$

with $[a, a^\dagger] = 1$, we can simplify (4) as

$$\frac{d}{dt}\rho = \frac{\lambda}{2}(2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A). \tag{6}$$

Based on (5) we introduce a unitary squeezing operator

$$S = \exp\left[\frac{\psi}{2}(a^{\dagger 2}e^{-i\theta} - a^2 e^{i\theta})\right] = e^{-i\mathcal{N}\theta/2}\exp\left[\frac{\psi}{2}(a^{\dagger 2} - a^2)\right]e^{i\mathcal{N}\theta/2}, \tag{7}$$

where θ is a rotating angle, ψ is the squeezing parameter and $\mathcal{N} = a^\dagger a$. S causes the squeezing transform

$$\begin{aligned} A &= SaS^{-1} \\ &= e^{-i\mathcal{N}\theta/2}\exp\left[\frac{\psi}{2}(a^{\dagger 2} - a^2)\right]e^{i\mathcal{N}\theta/2}ae^{-i\mathcal{N}\theta/2}\exp\left[-\frac{\psi}{2}(a^{\dagger 2} - a^2)\right]e^{i\mathcal{N}\theta/2} \\ &= a\cosh\psi - a^\dagger \sinh\psi e^{-i\theta}. \end{aligned} \tag{8}$$

Setting

$$\rho' = S^{-1}\rho S, \tag{9}$$

and making the squeezing transform for (6) we have the master equation for ρ' in terms of a and $a^\dagger$, i.e.

$$\frac{d}{dt}\rho' = \frac{\lambda}{2}(2a\rho' a^\dagger - a^\dagger a\rho' - \rho' a^\dagger a). \tag{10}$$

This simplification from (1) to (10) by the squeezing transform, so far as our knowledge concerned, has not been reported in the literature before.

Equation (10) has a similar form as the first part $-\frac{\lambda}{2}(N+1)[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a]$ of the right-hand side in (1) when $N = 0$, the latter represents the transfer through the decay of photons from the quantum system to the heat bath during the damping. In Sect. 3 we shall introduce the entangled state representation to convert (10) into a c-number equation.

3 Brief Review of the Entangled State Representation

Takahashi and Umezawa in [5] introduced Thermo Field Dynamics (TFD) to convert the statistical average at nonzero temperature T into equivalent pure state expectation value at the expense of introducing an auxilary freedom: For each real field space $\mathcal{H}$ they introduced a fictitious field (or a so-called tilde-conjugate field) $\tilde{\mathcal{H}}$. Thus the real vacuum state $|0\rangle$ in $\mathcal{H}$ has been doubled to $|0,\tilde{0}\rangle$ in $\mathcal{H}\otimes\tilde{\mathcal{H}}$. Similarly, the creation operator $a^\dagger$ and annihilation operator a acting on $\mathcal{H}$, are accompanied with a (or $a^\dagger$) in $\tilde{\mathcal{H}}$. In [6, 7, 12–14] we have introduced the entangled state $|\eta\rangle$,

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^*\tilde{a}^\dagger + a^\dagger\tilde{a}^\dagger\right)|0,\tilde{0}\rangle, \quad \eta = \eta_1 + i\eta_2. \tag{11}$$

$|\eta\rangle$ is the common eigenvector of $(a - \tilde{a}^\dagger)$ and $(\tilde{a} - a^\dagger)$, i.e.

$$\begin{aligned} &(a - \tilde{a}^\dagger)|\eta\rangle = \eta|\eta\rangle, \qquad (\tilde{a} - a^\dagger)|\eta\rangle = -\eta^*|\eta\rangle, \\ &\langle\eta|(a^\dagger - \tilde{a}) = \eta^*\langle\eta|, \qquad \langle\eta|(\tilde{a}^\dagger - a) = -\eta\langle\eta|, \end{aligned} \tag{12}$$

then we see

$$\begin{aligned} &\langle\eta'(a^\dagger - \tilde{a})|\eta\rangle = \eta'^*\langle\eta'|\eta\rangle = \eta^*\langle\eta'|\eta\rangle, \\ &\langle\eta'(a - \tilde{a}^\dagger)|\eta\rangle = \eta'\langle\eta'|\eta\rangle = \eta\langle\eta'|\eta\rangle, \end{aligned} \tag{13}$$

which leads to the orthonormal property $\langle\eta'|\eta\rangle = \pi\delta(\eta'-\eta)\delta(\eta'^*-\eta^*)$. Using the technique of integration within an ordered product (IWOP) of operators [6–9], we can prove the completeness relation of $|\eta\rangle$,

$$\int \frac{d^2\eta}{\pi}|\eta\rangle\langle\eta| = 1. \tag{14}$$

$|\eta\rangle$ can also be expressed as

$$|\eta\rangle = D(\eta)|I\rangle, \tag{15}$$

where

$$D(\eta) = e^{\eta a^\dagger - \eta^* a} \tag{16}$$

is the displacement operator and

$$|I\rangle \equiv \exp(a^\dagger\tilde{a}^\dagger)|0,\tilde{0}\rangle = \sum_{n=0}^{\infty}|n,\tilde{n}\rangle \tag{17}$$

is the nonnormalizable limiting case of two-mode squeezed vacuum state, $|I\rangle = |\eta = 0\rangle$ in (11). Instead of using the P-representation [10, 11] (or Q-representation, or the Wigner function) approaches, we here transform (10) into its c-number equation by using the $|\eta\rangle$ representation. In fact, setting $\rho'|I\rangle \equiv |\rho'\rangle$, from (17) we have

$$a|I\rangle = \tilde{a}^\dagger|I\rangle; \qquad a^\dagger|I\rangle = \tilde{a}|I\rangle; \qquad a^\dagger a|I\rangle = \tilde{a}^\dagger\tilde{a}|I\rangle, \tag{18}$$

which leads to

$$\begin{aligned} D^\dagger(\eta)|I\rangle &= e^{-\frac{1}{2}|\eta|^2}e^{-\eta a^\dagger}e^{\eta^* a}|I\rangle = e^{-\frac{1}{2}|\eta|^2}e^{-\eta a^\dagger}e^{\eta^*\tilde{a}^\dagger}|I\rangle \\ &= e^{-\frac{1}{2}|\eta|^2}e^{\eta^*\tilde{a}^\dagger}e^{-\eta\tilde{a}}|I\rangle = \tilde{D}(\eta^*)|I\rangle. \end{aligned} \tag{19}$$

From (11) we can see

$$\begin{aligned} \langle\eta|\tilde{a}|\rho\rangle &= -\left(\frac{\partial}{\partial\eta}+\frac{\eta^*}{2}\right)\langle\eta|\rho\rangle, \qquad \langle\eta|a|\rho\rangle = \left(\frac{\partial}{\partial\eta^*}+\frac{\eta}{2}\right)\langle\eta|\rho\rangle, \\ \langle\eta|\tilde{a}^\dagger|\rho\rangle &= \left(\frac{\partial}{\partial\eta^*}-\frac{\eta}{2}\right)\langle\eta|\rho\rangle, \qquad \langle\eta|a^\dagger|\rho\rangle = -\left(\frac{\partial}{\partial\eta}-\frac{\eta^*}{2}\right)\langle\eta|\rho\rangle. \end{aligned} \tag{20}$$

Sandwitching (10) between $\langle\eta|$ and $|I\rangle$ and defining

$$F(t) = \langle\eta|\rho'\rangle = \langle\eta|\rho'(t)\exp(a^\dagger\tilde{a}^\dagger)|0,\tilde{0}\rangle, \tag{21}$$

then using (12) and (20) we obtain the following c-number equation

$$\frac{d}{dt}F(t) = -\frac{\lambda}{2}\langle\eta|(\eta^* a - \eta\tilde{a})|\rho'\rangle = -\frac{\lambda}{2}\left(\eta\frac{\partial}{\partial\eta}+\eta^*\frac{\partial}{\partial\eta^*}+\eta\eta^*\right)F(t). \tag{22}$$

Thus we conclude that the entangled state representation is also a good candidate for converting master equation into c-number equation.

4 Time Evolution of Density Operator ρ'

The success of transforming (1) to (10) brings convenience for deriving time evolution of the damped harmonic oscillator in squeezed bath. Equation (10) shows that the density operator ρ' evolves under the interference of the mode's a and $a^\dagger$. If the system is initially in the superposition of two coherent states, i.e.

$$\rho'(t=0) = |\Phi'(0)\rangle\langle\Phi'(0)| \equiv \mathcal{G}^2(|\alpha_1\rangle + |\alpha_2\rangle)(\langle\alpha_1| + \langle\alpha_2|), \tag{23}$$

where $\mathcal{G}$ is the normalization coefficient, then at time t the density operator is

$$\rho'(t) = \mathcal{G}^2 \sum_{i,j=1}^{2} |\alpha_i(t)\rangle\langle\alpha_j(t)| \equiv \mathcal{G}^2 \sum_{i,j=1}^{2} \rho'_{ij}, \tag{24}$$

where $\rho'_{ij} = |\alpha_i(t)\rangle\langle\alpha_j(t)|$, i, $j = 1, 2$, and damping of $\alpha_i(t)$ is

$$\alpha_1(t) = \alpha_1 e^{-\gamma t}, \qquad \alpha_2(t) = \alpha_2 e^{-\gamma t}, \tag{25}$$

γ will be determined shortly later. Letting $F_{ij}(t) = \langle\eta|\rho'_{ij}\rangle$, $|\rho'_{ij}\rangle = \rho'_{ij}|I\rangle$, noticing $\langle\alpha_j(t)|a^\dagger = \alpha_j^*(t)\langle\alpha_j(t)|$, and using

$$|\alpha_i(t)\rangle\langle\alpha_j(t)|\tilde{a}|I\rangle = |\alpha_i(t)\rangle\langle\alpha_j(t)|a^\dagger|I\rangle = \alpha_j^*(t)|\alpha_i(t)\rangle\langle\alpha_j(t)|I\rangle = \alpha_j^*(t)|\rho'_{ij}\rangle, \tag{26}$$

by substituting (24) into (22), we have

$$\begin{aligned} \frac{d}{dt}F_{ij}(t) &= \frac{d}{dt}\langle\eta|\rho'_{ij}\rangle = -\frac{\lambda}{2}\langle\eta|(\eta^* a - \eta\tilde{a})\rho'_{ij}|I\rangle \\ &= -\frac{\lambda}{2}\langle\eta|(\eta^* a - \eta\tilde{a})|\alpha_i(t)\rangle\langle\alpha_j(t)|I\rangle \\ &= -\frac{\lambda}{2}[\eta^*\alpha_i(t) - \eta\alpha_j^*(t)]F_{ij}(t). \end{aligned} \tag{27}$$

Its solution is

$$F_{ij}(t) = C_{ij}\exp\left[\frac{\lambda}{2\gamma}(\eta^*\alpha_i - \eta\alpha_j^*)e^{-\gamma t}\right], \tag{28}$$

where C_{ij} is the coefficient determined by (21) and (28) at $t = 0$, i.e.

$$C_{ij} = F_{ij}(0)\exp\left[-\frac{\lambda}{2\gamma}(\eta^*\alpha_i - \eta\alpha_j^*)\right]. \tag{29}$$

In case of $i = j = 1$,

$$\begin{aligned} C_{11} &= F_{11}(0)\exp\left[-\frac{\lambda}{2\gamma}(\eta^*\alpha_1 - \eta\alpha_1^*)\right] \\ &= \langle\eta|\alpha_1\rangle\langle\alpha_1|I\rangle\exp\left[-\frac{\lambda}{2\gamma}(\eta^*\alpha_1 - \eta\alpha_1^*)\right], \end{aligned} \tag{30}$$

from (19) and $\langle 0|I\rangle = \sum_{n=0}^{\infty}\langle 0|n, \tilde{n}\rangle = |\tilde{0}\rangle$, we have

$$|\rho'_{11}\rangle = |\alpha_1\rangle\langle\alpha_1|I\rangle = D(\alpha_1)|0\rangle\langle 0|D^\dagger(\alpha_1)|I\rangle = D(\alpha_1)\tilde{D}(\alpha_1^*)|0\rangle\langle 0|I\rangle$$
$$= D(\alpha_1)\tilde{D}(\alpha_1^*)|0\rangle|\tilde{0}\rangle = |\alpha_1, \tilde{\alpha}_1^*\rangle, \tag{31}$$

therefore

$$C_{11} \equiv \langle\eta|\alpha_1, \tilde{\alpha}_1^*\rangle \exp\left[-\frac{\lambda}{2\gamma}(\eta^*\alpha_1 - \eta\alpha_1^*)\right]$$
$$= \langle 0, \tilde{0}| \exp\left(-\frac{1}{2}|\eta|^2 + \eta^* a - \eta\tilde{a} + a\tilde{a}\right)|\alpha_1, \tilde{\alpha}_1^*\rangle \exp\left[-\frac{\lambda}{2\gamma}(\eta^*\alpha_1 - \eta\alpha_1^*)\right]$$
$$= \exp\left[-\frac{1}{2}|\eta|^2 + \left(1 - \frac{\lambda}{2\gamma}\right)\eta^*\alpha_1 - \left(1 - \frac{\lambda}{2\gamma}\right)\eta\alpha_1^*\right]. \tag{32}$$

Using (14) and (15) we can see

$$|\rho'_{11}\rangle = \rho'_{11}|I\rangle = \frac{1}{\pi}\int d^2\eta|\eta\rangle\langle\eta|\rho'_{11}\rangle = \frac{1}{\pi}\int d^2\eta\langle\eta|\rho'_{11}\rangle D(\eta)|I\rangle. \tag{33}$$

Since the state $|I\rangle$ has nothing to do with the integration over $d^2\eta$, so we have

$$\rho'_{11} = \int \frac{d^2\eta}{\pi}\langle\eta|\rho'_{11}\rangle D(\eta) + w, \tag{34}$$

where the w operator satisfies the following constraint

$$w|I\rangle = we^{a^\dagger\tilde{a}^\dagger}|0, \tilde{0}\rangle = w\sum_{n=0}^{\infty}|n, \tilde{n}\rangle = 0, \tag{35}$$

the solution to (35) must include "∼" (tilde) mode, for example $w = a\tilde{a}e^{-a^\dagger\tilde{a}^\dagger}$, but we only need real mode solution as density operator, so we can neglect w from physical consideration, therefore we extract $|I\rangle$ from (33) and then using (16) to obtain the manifest form of ρ'_{11} (only real mode)

$$\rho'_{11} = \int \frac{d^2\eta}{\pi}\langle\eta|\rho'_{11}\rangle D(\eta)$$
$$= \int \frac{d^2\eta}{\pi}\exp\left[-|\eta|^2 + \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma}e^{-\gamma t}\right)\alpha_1\eta^*\right.$$
$$\left. - \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma}e^{-\gamma t}\right)\alpha_1^*\eta\right] : e^{\eta a^\dagger}e^{-\eta^* a} :$$
$$= \int \frac{d^2\eta}{\pi} : \exp\left\{-|\eta|^2 + \left[a^\dagger - \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma}e^{-\gamma t}\right)\alpha_1^*\right]\eta\right.$$
$$\left. + \left[-a + \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma}e^{-\gamma t}\right)\alpha_1\right]\eta^*\right\} :$$
$$= : \exp\left[-a^\dagger a + \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma}e^{-\gamma t}\right)(\alpha_1 a^\dagger + \alpha_1^* a)\right.$$
$$\left. - \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma}e^{-\gamma t}\right)^2|\alpha_1|^2\right] : . \tag{36}$$

In reference to $|z\rangle\langle z| =: \exp(-|z|^2 + za^\dagger + z^* a - a^\dagger a) :$, we can put (36) as the coherent state projector

$$\rho_{11}' = \left|\left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)\alpha_1\right\rangle\left\langle\left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)\alpha_1\right|. \tag{37}$$

Comparing (37) with $\rho_{11}' = |\alpha_1 e^{-\gamma t}\rangle\langle\alpha_1 e^{-\gamma t}|$, we have

$$\frac{\lambda}{2\gamma} = 1, \tag{38}$$

thus

$$\rho_{11}' = \left|\alpha_1 e^{-\frac{\lambda}{2}t}\right\rangle\left\langle\alpha_1 e^{-\frac{\lambda}{2}t}\right|. \tag{39}$$

Similarly,

$$\begin{gathered} \rho_{22}' = \left|\alpha_2 e^{-\frac{\lambda}{2}t}\right\rangle\left\langle\alpha_2 e^{-\frac{\lambda}{2}t}\right|, \\ C_{22} = \exp\left[-\frac{1}{2}|\eta|^2 + \left(1 - \frac{\lambda}{2\gamma}\right)\eta^*\alpha_2 - \left(1 - \frac{\lambda}{2\gamma}\right)\eta\alpha_2^*\right], \quad \frac{\lambda}{2\gamma} = 1. \end{gathered} \tag{40}$$

When $i = 1,\ j = 2$, we have

$$C_{12} \equiv \exp\left[-\frac{1}{2}|\eta|^2 + \left(1 - \frac{\lambda}{2\gamma}\right)\eta^*\alpha_1 - \left(1 - \frac{\lambda}{2\gamma}\right)\eta\alpha_2^* + \alpha_1\alpha_2^* - \frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2}\right], \tag{41}$$

thus

$$\begin{aligned} \rho_{12}' &= \int \frac{d^2\eta}{\pi} \langle\eta|\rho_{12}'\rangle D(\eta) \\ &= \int \frac{d^2\eta}{\pi} \exp\left[\alpha_1\alpha_2^* - \frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2}\right] \\ &\quad \times : \exp\left\{-|\eta|^2 + \left[a^\dagger - \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)\alpha_2^*\right]\eta \right. \\ &\quad \left. + \left[-a + \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)\alpha_1\right]\eta^*\right\} : \\ &= \left|\left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)\alpha_1\right\rangle\left\langle\left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)\alpha_2\right| \\ &\quad \times \exp\left\{\left[1 - \left(1 - \frac{\lambda}{2\gamma} + \frac{\lambda}{2\gamma} e^{-\gamma t}\right)^2\right]\left(-\frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2} + \alpha_1\alpha_2^*\right)\right\}. \end{aligned} \tag{42}$$

Comparing (42) with $\rho_{12}' = |\alpha_1 e^{-\gamma t}\rangle\langle\alpha_2 e^{-\gamma t}|$, we see $\frac{\lambda}{2\gamma} = 1$. Because the overlap of two coherent states is

$$\langle\alpha_2|\alpha_1\rangle = \exp\left[-\frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2} + \alpha_1\alpha_2^*\right], \tag{43}$$

we have

$$\rho_{12}' = \langle\alpha_2|\alpha_1\rangle^{(1-e^{-\lambda t})}\left|\alpha_1 e^{-\frac{\lambda}{2}t}\right\rangle\left\langle\alpha_2 e^{-\frac{\lambda}{2}t}\right|. \tag{44}$$

For $i = 2,\ j = 1$ we can derive

$$\rho'_{21} = \langle\alpha_1|\alpha_2\rangle^{(1-e^{-\lambda t})}\left|\alpha_2 e^{-\frac{\lambda}{2}t}\right\rangle\left\langle\alpha_1 e^{-\frac{\lambda}{2}t}\right|, \quad \frac{\lambda}{2\gamma} = 1$$
$$C_{21} = \exp\left[-\frac{1}{2}|\eta|^2 + \left(1 - \frac{\lambda}{2\gamma}\right)\eta^*\alpha_2 - \left(1 - \frac{\lambda}{2\gamma}\right)\eta\alpha_1^* + \alpha_2\alpha_1^* - \frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2}\right]. \tag{45}$$

Combining all the above results, we reach to

$$\rho'(t) = \mathcal{G}^2 \sum_{i,j=1}^{2} \langle\alpha_j|\alpha_i\rangle^{(1-e^{-\lambda t})}\left|\alpha_i e^{-\frac{\lambda}{2}t}\right\rangle\left\langle\alpha_j e^{-\frac{\lambda}{2}t}\right|. \tag{46}$$

5 The Solution to (1)

From (9) and (46) we know

$$\rho(t) = S\rho'(t)S^{-1} = \mathcal{G}^2 \sum_{i,j=1}^{2} \langle\alpha_j|\alpha_i\rangle^{(1-e^{-\lambda t})} S\left|\alpha_i e^{-\frac{\lambda}{2}t}\right\rangle\left\langle\alpha_j e^{-\frac{\lambda}{2}t}\right|S^{-1}, \tag{47}$$

where

$$\begin{aligned} S\left|\alpha_i e^{-\frac{\lambda}{2}t}\right\rangle &= e^{-i\mathcal{N}\theta/2}\exp\left[\frac{\psi}{2}(a^{\dagger 2} - a^2)\right]e^{i\mathcal{N}\theta/2}D\left(\alpha_i e^{-\frac{\lambda}{2}t}\right)|0\rangle \\ &= \operatorname{sec}h^{1/2}\psi\, D\left(\alpha_i e^{-\frac{\lambda}{2}t}\cosh\psi + \alpha_i^* e^{-\frac{\lambda}{2}t}e^{-i\theta}\sinh\psi\right)\exp\left[\frac{a^{\dagger 2}}{2}e^{-i\theta}\tanh\psi\right]|0\rangle \end{aligned} \tag{48}$$

is a coherent squeezed state with displacement parameter $\varsigma = \alpha_i e^{-\frac{\lambda}{2}t}\cosh\psi + \alpha_i^* e^{-\frac{\lambda}{2}t}e^{-i\theta}$ $\sinh\psi$, and squeezing is determined by ψ and θ (related to N and M in (1)). Therefore, the final mixed state is a coherent squeezed state. This conclusion seems new.

In summary, by virtue of a squeezing transformation we have simplified the density operator equation (1) to (10). Employing the entangled state representation, we have converted (10) to its c-number equation in (22) and then derived the final result which turns out to be a mixed coherent squeezed states.

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